Perbandingan Dua Populasi 
(Bagian I)

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A comparative study, the common statistical term treatment is used to refer to the things that are being compared.

For comparing two treatments or populations, the two basic types of design are:

1. **Independent samples** (complete randomization).
2. **Matched pairs sample** (randomization within each matched pair).
Sampel Bebas
(Independent Samples)
Sampel Acak Bebas (*Independent*) dari Dua Populasi

<table>
<thead>
<tr>
<th>Sample</th>
<th>Summary Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1, X_2, \ldots, X_{n_1}$ from population 1</td>
<td>$\bar{X} = \frac{1}{n_1} \sum_{i=1}^{n_1} X_i$ \hspace{1cm} $S_1^2 = \frac{\sum_{i=1}^{n_1} (X_i - \bar{X})^2}{n_1 - 1}$</td>
</tr>
<tr>
<td>$Y_1, Y_2, \ldots, Y_{n_2}$ from population 2</td>
<td>$\bar{Y} = \frac{1}{n_2} \sum_{i=1}^{n_2} Y_i$ \hspace{1cm} $S_2^2 = \frac{\sum_{i=1}^{n_2} (Y_i - \bar{Y})^2}{n_2 - 1}$</td>
</tr>
</tbody>
</table>
**Large Samples Confidence Interval for $\mu_1 - \mu_2$**

When $n_1$ and $n_2$ are greater than 30, an approximate 100 $(1 - \alpha)$% confidence interval for $\mu_1 - \mu_2$ is given by

$$
\left( \bar{X} - \bar{Y} - z_{\alpha/2} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} , \quad \bar{X} - \bar{Y} + z_{\alpha/2} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \right)
$$

where $z_{\alpha/2}$ is the upper $\alpha/2$ point of $N(0, 1)$. 

Sampel Acak Bebas (Independent) dari Dua Populasi

Testing $H_0: \mu_1 - \mu_2 = \delta_0$ with Large Samples

Test statistic:

$$Z = \frac{\bar{X} - \bar{Y} - \delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

<table>
<thead>
<tr>
<th>Alternative Hypothesis</th>
<th>Level $\alpha$ Rejection Region</th>
</tr>
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<tr>
<td>$H_1: \mu_1 - \mu_2 &gt; \delta_0$</td>
<td>$R: Z \geq z_\alpha$</td>
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Large Samples Test with a One-Sided Alternative

In June two years ago, chemical analyses were made of 85 water samples (each of unit volume) taken from various parts of a city lake, and the measurements of chlorine content were recorded. During the next two winters, the use of road salt was substantially reduced in the catchment areas of the lake. This June, 110 water samples were analyzed and their chlorine contents recorded. Calculations of the mean and the standard deviation for the two sets of data give

<table>
<thead>
<tr>
<th>Chlorine Content</th>
<th>Two Years Ago</th>
<th>Current Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>18.3</td>
<td>17.8</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.2</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Test the claim that lower salt usage has reduced the amount of chlorine in the lake. Base your decision on the $P$-value.
Let $\mu_1$ be the population mean two years ago and $\mu_2$ the population mean in the current year. Because the claim is that $\mu_2$ is less than $\mu_1$, we formulate the hypotheses

$$H_0 : \mu_1 - \mu_2 = 0 \quad \text{versus} \quad H_1 : \mu_1 - \mu_2 > 0$$

With the test statistic

$$Z = \frac{\overline{X} - \overline{Y}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

the rejection region should be of the form $R : Z \geq c$ because $H_1$ is right-sided.
Using the data

\[ n_1 = 85 \quad \bar{x} = 18.3 \quad s_1 = 1.2 \]
\[ n_2 = 110 \quad \bar{y} = 17.8 \quad s_2 = 1.8 \]

we calculate

\[ z = \frac{18.3 - 17.8}{\sqrt{\frac{(1.2)^2}{85} + \frac{(1.8)^2}{110}}} = \frac{.5}{.2154} = 2.32 \]

The significance probability of this observed value is (see Figure 4)

\[ P\text{–value} = P[Z \geq 2.32] = .0102 \]

Because the \( P\text{–value} \) is very small, we conclude that there is strong evidence to reject \( H_0 \) and support \( H_1 \).
Sampel Acak Bebas (Independent) dari Dua Populasi
(Sampel Kecil: Populasi Normal dengan Ragam Sama)

Additional Assumptions When the Sample Sizes Are Small
1. Both populations are normal.
2. The population standard deviations $\sigma_1$ and $\sigma_2$ are equal.

Pooled Estimator of the Common $\sigma^2$

\[
S_{pooled}^2 = \frac{\sum_{i=1}^{n_1} (X_i - \bar{X})^2 + \sum_{i=1}^{n_2} (Y_i - \bar{Y})^2}{n_1 + n_2 - 2}
\]

\[
= \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}
\]
Sampel Acak Bebas (Independent) dari Dua Populasi (Sampel Kecil: Populasi Normal dengan Ragam Sama)

Confidence Interval for $\mu_1 - \mu_2$ Small Samples and $\sigma_1 = \sigma_2$

A 100 $(1 - \alpha)\%$ confidence interval for $\mu_1 - \mu_2$ is given by

$$\bar{X} - \bar{Y} \pm t_{\alpha/2} S_{\text{pooled}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where

$$S_{\text{pooled}}^2 = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}$$

and $t_{\alpha/2}$ is the upper $\alpha/2$ point of the $t$ distribution with
d.f. $= n_1 + n_2 - 2$. 
Sampel Acak Bebas (*Independent*) dari Dua Populasi
(Sampel Kecil: Populasi Normal dengan Ragam Sama)

Testing \( H_0 : \mu_1 - \mu_2 = \delta_0 \) with Small Samples and \( \sigma_1 = \sigma_2 \)

Test statistic:

\[
T = \frac{\bar{X} - \bar{Y} - \delta_0}{S_{\text{pooled}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad \text{d.f.} = n_1 + n_2 - 2
\]

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Testing the Equality of Mean Computer Anxiety Scores

Refer to the computer anxiety scores (CARS) described in Example 9 and the summary statistics

Female CARS

\[ n_1 = 15 \quad \bar{x} = 2.514 \quad s_1 = .773 \]

Male CARS

\[ n_2 = 20 \quad \bar{y} = 2.963 \quad s_2 = .525 \]

Do these data strongly indicate that the mean score for females is lower than that for males? Test at level \( \alpha = .05 \).
We are seeking strong evidence in support of the hypothesis that the mean computer anxiety score for females ($\mu_1$) is less than the mean score for males. Therefore the alternative hypothesis should be taken as $H_1: \mu_1 < \mu_2$ or $H_1: \mu_1 - \mu_2 < 0$, and our problem can be stated as testing

$$H_0: \mu_1 - \mu_2 = 0 \quad \text{versus} \quad H_1: \mu_1 - \mu_2 < 0$$

We employ the test statistic

$$T = \frac{\bar{X} - \bar{Y}}{S_{\text{pooled}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad \text{d.f.} = n_1 + n_2 - 2$$

and set the left-sided rejection region $R: T \leq -t_{.05}$. For d.f. = $n_1 + n_2 - 2 = 33$, we approximate the tabled value as $t_{.05} = 1.692$, so the rejection region is $R: T \leq -1.692$. 
With $S_{\text{pooled}} = .642$ already calculated in Example 9, the observed value of the test statistic $T$ is

$$t = \frac{2.514 - 2.963}{\sqrt{\frac{1}{15} + \frac{1}{20}}} = \frac{-0.449}{0.2193} = -2.05$$

This value lies in the rejection region $R$. Consequently, at the .05 level of significance, we reject the null hypothesis in favor of the alternative hypothesis that males have a higher mean computer anxiety score.

A computer calculation gives a $P-$value of about .025 so the evidence of $H_1$ is moderately strong.
Materi Responsi
10.8 A linguist wants to compare the writing styles in two magazines and one measure is the number of words per sentence. On the basis of 50 randomly selected sentences from each source, she finds

Magazine 1 \( n_1 = 50 \quad \bar{x} = 12.6 \quad s_1 = 4.2 \)

Magazine 2 \( n_2 = 50 \quad \bar{y} = 9.5 \quad s_2 = 1.9 \)

Determine a 98% confidence interval for the difference in mean number of words per sentence.
10.9 Refer to Exercise 10.8. Perform a test of hypothesis that is intended to show that the mean for magazine 1 is more than 2 words larger than the mean for magazine 2.

(a) Formulate the null and alternative hypotheses. (Define any symbols you use.)

(b) State the test statistic and the rejection region with $\alpha = .05$.

(c) Perform the test at $\alpha = .05$. Also, find the $P$-value and comment.
In a study of interspousal aggression and its possible effect on child behavior, the behavior problem checklist (BPC) scores were recorded for 47 children whose parents were classified as aggressive. The sample mean and standard deviation were 7.92 and 3.45, respectively. For a sample of 38 children whose parents were classified as nonaggressive, the mean and standard deviation of the BPC scores were 5.80 and 2.87, respectively. Do these observations substantiate the conjecture that the children of aggressive families have a higher mean BPC than those of nonaggressive families? (Answer by calculating the $P$-value.)
10.16 The data on the weight (lb) of male and female wolves, from Table D.9 of the Data Bank, are

<table>
<thead>
<tr>
<th>Female</th>
<th>57</th>
<th>84</th>
<th>90</th>
<th>71</th>
<th>71</th>
<th>77</th>
<th>68</th>
<th>73</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>71</td>
<td>93</td>
<td>101</td>
<td>84</td>
<td>88</td>
<td>117</td>
<td>86</td>
<td>86</td>
</tr>
</tbody>
</table>

(a) Test the null hypothesis that the mean weights of males and females are equal versus a two-sided alternative. Take $\alpha = .05$.

(b) Obtain a 95% confidence interval for the difference of population mean weights.

(c) State any assumptions you make about the populations.
Pustaka


- Pustaka lain yang relevan.
Catatan Kuliah

Bisa di-download di

kusmansadik.wordpress.com
Terima Kasih