Analisis Statistika (STK511)

Analisis Regresi

(Bagian I)

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In many investigations, two or more variables are observed for each experimental unit in order to determine:

1. Whether the variables are related.
2. How strong the relationships appear to be.
3. Whether one variable of primary interest can be predicted from observations on the others.
Regression analysis concerns the study of relationships between quantitative variables with the object of identifying, estimating, and validating the relationship.

The estimated relationship can then be used to predict one variable from the value of the other variable(s).

Studies of relation among variables abound in virtually all disciplines of science and the humanities.
A regression problem involving a single predictor (also called simple regression) arises when we wish to study the relation between two variables x and y and use it to predict y from x.

The variable x acts as an independent variable whose values are controlled by the experimenter.

The variable y depends on x and is also subjected to unaccountable variations or errors.
Regresi Linear Sederhana

Notation

\[ x = \text{independent variable, also called predictor variable, explanatory variable, causal variable, or input variable} \]

\[ y = \text{dependent or response variable} \]
Relief from Symptoms of Allergy Related to Dosage

In one stage of the development of a new drug for an allergy, an experiment is conducted to study how different dosages of the drug affect the duration of relief from the allergic symptoms. Ten patients are included in the experiment. Each patient receives a specified dosage of the drug and is asked to report back as soon as the protection of the drug seems to wear off. The observations are recorded in Table 1, which shows the dosage $x$ and duration of relief $y$ for the 10 patients.
**TABLE 1** Dosage $x$ (in Milligrams) and the Number of Hours of Relief $y$ from Allergy for Ten Patients

<table>
<thead>
<tr>
<th>Dosage $x$</th>
<th>Duration of Relief $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
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<td>14</td>
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<td>6</td>
<td>16</td>
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<td>7</td>
<td>22</td>
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<tr>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>9</td>
<td>22</td>
</tr>
</tbody>
</table>
First Step in the Analysis

Plotting a scatter diagram is an important preliminary step prior to undertaking a formal statistical analysis of the relationship between two variables.

Figure 1  Scatter diagram of the data of Table 1.
Figure 2   Graph of straight line $y = \beta_0 + \beta_1 x$. 
Statistical Model for a Straight Line Regression

We assume that the response \( Y \) is a random variable that is related to the input variable \( x \) by

\[
Y_i = \beta_0 + \beta_1 x_i + e_i \quad i = 1, \ldots, n
\]

where:

1. \( Y_i \) denotes the response corresponding to the \( i \)th experimental run in which the input variable \( x \) is set at the value \( x_i \).
2. \( e_1, \ldots, e_n \) are the unknown error components that are superimposed on the true linear relation. These are unobservable random variables, which we assume are independently and normally distributed with mean zero and an unknown standard deviation \( \sigma \).
3. The parameters \( \beta_0 \) and \( \beta_1 \), which together locate the straight line, are unknown.
Interpretasi Parameter Regresi

$\beta_0$ : Nilai $y$ pada saat $x$ bernilai 0. (Perhatikan, pada beberapa keadaan, parameter ini tidak bisa diinterpretasikan. Mengapa?)

$\beta_1$ : Besarnya perubahan $y$ apabila $x$ berubah sebesar satu satuan unit
Metode Kuadrat Terkecil (The Method of Least Squares)

\[ d_i = (y_i - b_0 - b_1 x_i) \]

\[ y = b_0 + b_1 x \]

\[ D = \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2 \]

Considering such discrepancies at all the \( n \) points, we take

\[ D = \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2 \]

as an overall measure of the discrepancy of the observed points from the trial line \( y = b_0 + b_1 x \). The magnitude of \( D \) obviously depends on the line that is drawn. In other words, it depends on \( b_0 \) and \( b_1 \), the two quantities that determine the trial line. A good fit will make \( D \) as small as possible. We now state the principle of least squares in general terms to indicate its usefulness to fitting many other models.
Metode Kuadrat Terkecil (*The Method of Least Squares*)

**The Principle of Least Squares**

Determine the values for the parameters so that the overall discrepancy

\[ D = \sum (\text{Observed response} - \text{Predicted response})^2 \]

is minimized.

The parameter values thus determined are called the **least squares estimates**.

The particular values \( b_0 \) and \( b_1 \) that minimize the sum of squares are denoted by \( \hat{\beta}_0 \) and \( \hat{\beta}_1 \), respectively. The ^ over a parameter indicates that it is an estimate of the parameter. They are called the **least squares estimates** of the regression parameters \( \beta_0 \) and \( \beta_1 \). The **best fitting straight line** or **best fitting regression line** is then given by the equation

\[ \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \]

where the hat over \( y \) indicates that it is an estimated quantity.
### Basic Notation

\[
\bar{x} = \frac{1}{n} \sum x \quad \bar{y} = \frac{1}{n} \sum y
\]

\[
S_{xx} = \sum (x - \bar{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}
\]

\[
S_{yy} = \sum (y - \bar{y})^2 = \sum y^2 - \frac{(\sum y)^2}{n}
\]

\[
S_{xy} = \sum (x - \bar{x})(y - \bar{y}) = \sum xy - \frac{(\sum x)(\sum y)}{n}
\]
Least squares estimator of $\beta_0$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Least squares estimator of $\beta_1$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$$

Fitted (or estimated) regression line

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

Residuals

$$\hat{e}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i \quad i = 1, \ldots, n$$
The residual sum of squares or the sum of squares due to error is

$$SSE = \sum_{i=1}^{n} \hat{e}_i^2 = S_{yy} - \frac{S_{xy}^2}{S_{xx}}$$

**Sum of Squares due to Error (SSE) ---- Jumlah Kuadrat Galat (JKG)**

**Estimate of Variance**

The estimator of the error variance $\sigma^2$ is

$$S^2 = \frac{SSE}{n - 2}$$
**TABLE 3**  Computations for the Least Squares Line, SSE, and Residuals Using the Data of Table 1

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$x^2$</th>
<th>$y^2$</th>
<th>$xy$</th>
<th>$\hat{\beta}_0 + \hat{\beta}_1 x$</th>
<th>Residual $\hat{e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9</td>
<td>9</td>
<td>81</td>
<td>27</td>
<td>7.15</td>
<td>1.85</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>9</td>
<td>25</td>
<td>15</td>
<td>7.15</td>
<td>-2.15</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>16</td>
<td>144</td>
<td>48</td>
<td>9.89</td>
<td>2.11</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>25</td>
<td>81</td>
<td>45</td>
<td>12.63</td>
<td>-3.63</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>36</td>
<td>196</td>
<td>84</td>
<td>15.37</td>
<td>-1.37</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
<td>36</td>
<td>256</td>
<td>96</td>
<td>15.37</td>
<td>0.63</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
<td>49</td>
<td>484</td>
<td>154</td>
<td>18.11</td>
<td>3.89</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
<td>64</td>
<td>324</td>
<td>144</td>
<td>20.85</td>
<td>-2.85</td>
</tr>
<tr>
<td>8</td>
<td>24</td>
<td>64</td>
<td>576</td>
<td>192</td>
<td>20.85</td>
<td>3.15</td>
</tr>
<tr>
<td>9</td>
<td>22</td>
<td>81</td>
<td>484</td>
<td>198</td>
<td>23.59</td>
<td>-1.59</td>
</tr>
</tbody>
</table>

**Total** 59 151 389 2651 1003

$\bar{x} = 5.9, \quad \bar{y} = 15.1$

$\hat{\beta}_1 = \frac{112.1}{40.9} = 2.74$

$\hat{\beta}_0 = 15.1 - 2.74 \times 5.9 = -1.07$

$S_{xx} = 389 - \frac{(59)^2}{10} = 40.9$

$S_{yy} = 2651 - \frac{(151)^2}{10} = 370.9$

$S_{xy} = 1003 - \frac{59 \times 151}{10} = 112.1$

$SSE = 370.9 - \frac{(112.1)^2}{40.9} = 63.6528$

.04 (rounding error)
The equation of the line fitted by the least squares method is then

\[ \hat{y} = -1.07 + 2.74x \]
The residuals $\hat{e}_i = y_i - \hat{y}_i = y_i + 1.07 - 2.74 x_i$ are computed in the last column of Table 3. The sum of squares of the residuals is

$$\sum_{i=1}^{n} \hat{e}_i^2 = (1.85)^2 + (-2.15)^2 + (2.11)^2 + \cdots + (-1.59)^2 = 63.653$$

The estimate of the variance $\sigma^2$ is

$$s^2 = \frac{\text{SSE}}{n - 2} = \frac{63.6528}{8} = 7.96$$

$s = 2.821$
1. The standard deviations (also called standard errors) of the least squares estimators are

\[ S.E. (\hat{\beta}_1) = \frac{\sigma}{\sqrt{S_{xx}}} \]

\[ S.E. (\hat{\beta}_0) = \sigma \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}} \]

To estimate the standard error, use

\[ S = \sqrt{\frac{SSE}{n - 2}} \] in place of \( \sigma \)
2. Inferences about the slope $\beta_1$ are based on the $t$ distribution

$$T = \frac{\hat{\beta}_1 - \beta_1}{S/\sqrt{S_{xx}}} \quad \text{d.f.} = n - 2$$

Inferences about the intercept $\beta_0$ are based on the $t$ distribution

$$T = \frac{\hat{\beta}_0 - \beta_0}{S \sqrt{\frac{1}{n} + \frac{x^2}{S_{xx}}} \quad \text{d.f.} = n - 2}$$
Inferensia untuk slope ($\beta_1$)

In a regression analysis problem, it is of special interest to determine whether the expected response does or does not vary with the magnitude of the input variable $x$. According to the linear regression model,

$$\text{Expected response} = \beta_0 + \beta_1 x$$

This does not change with a change in $x$ if and only if $\beta_1 = 0$. We can therefore test the null hypothesis $H_0 : \beta_1 = 0$ against a one- or a two-sided alternative, depending on the nature of the relation that is anticipated. If we refer to the boxed statement (2) of Section 5, the null hypothesis $H_0 : \beta_1 = 0$ is to be tested using the test statistic

$$T = \frac{\hat{\beta}_1}{S/\sqrt{S_{xx}}} \quad \text{d.f.} = n - 2$$
Inferensia untuk slope ($\beta_1$)

A Test to Establish That Duration of Relief Increases with Dosage

Do the data given in Table 1 constitute strong evidence that the mean duration of relief increases with higher dosages of the drug?

For an increasing relation, we must have $\beta_1 > 0$. Therefore, we are to test the null hypothesis $H_0 : \beta_1 = 0$ versus the one-sided alternative $H_1 : \beta_1 > 0$. We select $\alpha = .05$. Since $t_{.05} = 1.860$, with d.f. = 8 we set the rejection region $R : T \geq 1.860$. Using the calculations that follow Table 3, we have

$$\hat{\beta}_1 = 2.74$$

$$s^2 = \frac{\text{SSE}}{n - 2} = \frac{63.6528}{8} = 7.9566, \quad s = 2.8207$$
Inferensia untuk slope ($\beta_1$)

Estimated S.E.($\hat{\beta}_1$) = \[ \frac{s}{\sqrt{S_{xx}}} = \frac{2.8207}{\sqrt{40.90}} = .441 \]

Test statistic \[ t = \frac{2.74}{.441} = 6.213 \]

The observed $t$ value is in the rejection region, so $H_0$ is rejected. Moreover, 6.213 is much larger than $t_{.005} = 3.355$, so the $P$–value is much smaller than .005.

A computer calculation gives $P[T > 6.213] = .0001$. There is strong evidence that larger dosages of the drug tend to increase the duration of relief over the range covered in the study.
Inferensi untuk slope ($\beta_1$)

More generally, we may test whether or not $\beta_1$ is equal to some specified value $\beta_{10}$, not necessarily zero.

The test of the null hypothesis

$$H_0: \beta_1 = \beta_{10}$$

is based on

$$T = \frac{\hat{\beta}_1 - \beta_{10}}{S/\sqrt{S_{xx}}} \quad \text{d.f.} = n - 2$$

A $100(1 - \alpha)\%$ confidence interval for $\beta_1$ is

$$\left( \hat{\beta}_1 - t_{\alpha/2} \frac{S}{\sqrt{S_{xx}}}, \hat{\beta}_1 + t_{\alpha/2} \frac{S}{\sqrt{S_{xx}}} \right)$$

where $t_{\alpha/2}$ is the upper $\alpha/2$ point of the $t$ distribution with d.f. = $n - 2$. 
**Inferensia untuk intersep \((\beta_0)\)**

A 100\((1 - \alpha)\)% **confidence interval** for \(\beta_0\) is

\[
\left( \hat{\beta}_0 - t_{\alpha/2} S \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}} , \quad \hat{\beta}_0 + t_{\alpha/2} S \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}} \right)
\]

To illustrate this formula, let us consider the data of Table 1. In Table 3, we have found \(\hat{\beta}_0 = -1.07\), \(\bar{x} = 5.9\), and \(S_{xx} = 40.9\). Also, \(s = 2.8207\). Therefore, a 95% confidence interval for \(\beta_0\) is calculated as

\[
-1.07 \pm 2.306 \times 2.8207 \sqrt{\frac{1}{10} + \frac{(5.9)^2}{40.9}}
\]

\[
= -1.07 \pm 6.34 \quad \text{or} \quad (-7.41, 5.27)
\]

Note that \(\beta_0\) represents the mean response corresponding to the value 0 for the input variable \(x\). In the drug evaluation problem of Example 4, the parameter \(\beta_0\) is of little practical interest because the range of \(x\) values covered in the experiment was 3 to 9 and it would be unrealistic to extend the line to \(x = 0\). In fact, the estimate \(\hat{\beta}_0 = -1.07\) does not have an interpretation as a (time) duration of relief.
ESTIMATION OF THE MEAN RESPONSE FOR A SPECIFIED $x$ VALUE

A 100$(1 - \alpha)$% confidence interval for the expected response $\beta_0 + \beta_1 x^*$ is

$$\hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{\alpha/2} S \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{s_{xx}}}$$

To test the hypothesis that $\beta_0 + \beta_1 x^* = \mu_0$, some specified value, we use

$$T = \frac{\hat{\beta}_0 + \hat{\beta}_1 x^* - \mu_0}{s \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{s_{xx}}}} \quad \text{d.f.} = n - 2$$
A Confidence Interval for the Expected Duration of Relief

Again consider the data given in Table 1 and the calculations for the regression analysis given in Table 3. Obtain a 95% confidence interval for the expected duration of relief when the dosage is (a) $x^* = 6$ and (b) $x^* = 9.5$. 
Inferensia untuk Penduga Respon Tunggal (Y)

PREDICTION OF A SINGLE RESPONSE FOR A SPECIFIED x VALUE

Suppose that we give a specified dosage $x^*$ of the drug to a single patient and we want to predict the duration of relief from the symptoms of allergy.

The estimated standard error when predicting a single observation $y$ at a given $x^*$ is

$$S \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}}$$
Calculating a Prediction Interval for a Future Trial

Once again, consider the drug trial data given in Table 1. A new trial is to be made on a single patient with the dosage \( x^* = 6.5 \) milligrams. Predict the duration of relief and give a 95% prediction interval for the duration of relief.

The predicted duration of relief is

\[
\hat{\beta}_0 + \hat{\beta}_1 x^* = -1.07 + 2.74 \times 6.5 = 16.74 \text{ hours}
\]

Since \( t_{0.025} = 2.306 \) with d.f. = 8, a 95% prediction interval for the new patient’s duration of relief is

\[
16.74 \pm 2.306 \times 2.8207 \sqrt{1 + \frac{1}{10} + \frac{(6.5 - 5.9)^2}{40.9}}
\]

\[
= 16.74 \pm 6.85 \quad \text{or} \quad (9.89, 23.59)
\]

This means we are 95% confident that this particular patient will have relief from symptoms of allergy for about 9.9 to 23.6 hours.
**Ilustrasi**: Lihat Johnson, *Example 9, hl. 464*

**Prediction after Fitting a Straight Line Relation of a Human Development Index to Internet Usage**

One measure of the development of a country is the Human Development Index (HDI) which combines life expectancy, literacy, educational attainment, and gross domestic product per capita into an index whose values lie between 0 and 1, inclusive.

We randomly selected fifteen countries, of the 152 countries, below the top twenty-five most developed countries on the list. HDI is the response variable $y$, and Internet usage per 100 persons, $x$, is the predictor variable. The data, given in Exercise 11.31, have the summary statistics

$$
n = 15 \quad \overline{x} = 9.953 \quad \overline{y} = .6670
$$

$$
S_{xx} = 1173.46 \quad S_{yy} = 20.471 \quad S_{xy} = .41772
$$
\[ n = 15 \quad \bar{x} = 9.953 \quad \bar{y} = 0.6670 \]
\[ S_{xx} = 1173.46 \quad S_{yy} = 20.471 \quad S_{xy} = 0.41772 \]

(a) Determine the equation of the best fitting straight line.

(b) Do the data substantiate the claim that Internet usage per 100 persons is a good predictor of HDI and that large values of both variables tend to occur together?

(c) Estimate the mean value of HDI for 18 Internet users per 100 persons and construct a 95% confidence interval.

(d) Find the predicted \( y \) for \( x = 43 \) Internet users per 100 persons.
Ukuran Kebaikan Model Regresi

As an index of how well the straight line model fits, it is then reasonable to consider the proportion of the y variability explained by the linear relation

\[ R^2 = \frac{\text{SS due to linear regression}}{\text{Total SS of } y} = \frac{S_{xy}^2 / S_{xx}}{S_{yy}} = \frac{S_{xy}^2}{S_{xx}S_{yy}} \]

is named the sample correlation coefficient. Thus, the square of the sample correlation coefficient represents the proportion of the y variability explained by the linear relation.
The strength of a linear relation is measured by

\[ R^2 = r^2 = \frac{S_{xy}^2}{S_{xx}S_{yy}} \]

which is the square of the sample correlation coefficient \( r \).

The value of \( r \) is always between \(-1\) and \(1\), inclusive whereas \( r^2 \) is always between \(0\) and \(1\).
The Proportion of Variability in Duration Explained by Dosage

Let us consider the drug trial data in Table 1. From the calculations provided in Table 3,

\[ S_{xx} = 40.9 \quad S_{yy} = 370.9 \quad S_{xy} = 112.1 \]

Fitted regression line

\[ \hat{y} = -1.07 + 2.74x \]

How much of the variability in \( y \) is explained by the linear regression model? To answer this question, we calculate

\[ R^2 = \frac{S_{xy}^2}{S_{xx}S_{yy}} = \frac{(112.1)^2}{40.9 \times 370.9} = .83 \]

This means that 83\% of the variability in \( y \) is explained by linear regression, and the linear model seems satisfactory in this respect.
Materi Responsi
11.50 Last week’s total number of hours worked by a student, \( y \), depends on the number of days, \( x \), he reported to work last week. Suppose the data from nine students provided:

\[
\begin{array}{c|cccccccc}
  x & 1 & 1 & 1 & 2 & 3 & 3 & 3 & 4 & 5 \\
  y & 8 & 6 & 7 & 10 & 15 & 12 & 13 & 19 & 18 \\
\end{array}
\]

(a) Plot the scatter diagram.

(b) Calculate \( \bar{x}, \bar{y}, S_{xx}, S_{yy}, \) and \( S_{xy} \).

(c) Determine the equation of the least squares fitted line and draw the line on the scatter diagram.

(d) Find the predicted number of hours \( y \) corresponding to \( x = 3 \) days.
Latihan Responsi (2) : Johnson (Exercise 11.53)

11.53 An experiment is conducted to determine how the strength $y$ of plastic fiber depends on the size $x$ of the droplets of a mixing polymer in suspension. Data of $(x, y)$ values, obtained from 15 runs of the experiment, have yielded the following summary statistics.

$$
\bar{x} = 8.3 \quad \bar{y} = 54.8 \\
S_{xx} = 5.6 \quad S_{xy} = -12.4 \quad S_{yy} = 38.7
$$

(a) Obtain the equation of the least squares regression line.

(b) Test the null hypothesis $H_0 : \beta_1 = -2$ against the alternative $H_1 : \beta_1 < -2$, with $\alpha = .05$.

(c) Estimate the expected fiber strength for droplet size $x = 10$ and set a 95% confidence interval.
11.55 A recent graduate moving to a new job collected a sample of monthly rent (dollars) and size (square feet) of 2-bedroom apartments in one area of a midwest city.

<table>
<thead>
<tr>
<th>Size</th>
<th>Rent</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>750</td>
</tr>
<tr>
<td>925</td>
<td>775</td>
</tr>
<tr>
<td>932</td>
<td>820</td>
</tr>
<tr>
<td>940</td>
<td>820</td>
</tr>
<tr>
<td>1000</td>
<td>850</td>
</tr>
<tr>
<td>1033</td>
<td>875</td>
</tr>
<tr>
<td>1050</td>
<td>915</td>
</tr>
<tr>
<td>1100</td>
<td>1040</td>
</tr>
</tbody>
</table>

(a) Plot the scatter diagram and find the least squares fit of a straight line.

(b) Do these data substantiate the claim that the monthly rent increases with the size of the apartment? (Test with \( \alpha = .05 \)).

(c) Give a 95% confidence interval for the expected increase in rent for one additional square foot.

(d) Give a 95% prediction interval for the monthly rent of a specific apartment having 1025 square feet.
11.62 Consider the data on all of the wolves in Table D.9 of the Data Bank concerning body length (cm) and weight (lb). Using Minitab or some other software program:

(a) Plot weight versus body length.

(b) Obtain the least squares fit of weight to the predictor variable body length.

(c) Test $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 > 0$ with $\alpha = .05$. 
### TABLE D.9  Physical Characteristics of Wolves

<table>
<thead>
<tr>
<th>Sex</th>
<th>Age</th>
<th>Weight (lb)</th>
<th>Body Length (cm)</th>
<th>Tail Length (cm)</th>
<th>Canine Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>4</td>
<td>71</td>
<td>134</td>
<td>44</td>
<td>28.7</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>57</td>
<td>123</td>
<td>46</td>
<td>27.0</td>
</tr>
<tr>
<td>F</td>
<td>4</td>
<td>84</td>
<td>129</td>
<td>49</td>
<td>27.2</td>
</tr>
<tr>
<td>M</td>
<td>4</td>
<td>93</td>
<td>143</td>
<td>46</td>
<td>30.5</td>
</tr>
<tr>
<td>M</td>
<td>4</td>
<td>101</td>
<td>148</td>
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<td>32.3</td>
</tr>
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<td>127</td>
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<td>25.8</td>
</tr>
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<td>2</td>
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<td>136</td>
<td>47</td>
<td>26.6</td>
</tr>
<tr>
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Pustaka

- Pustaka lain yang relevan.
Catatan Kuliah

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Terima Kasih